

$$\frac{\pi}{4} = 4 \arctan\left(\frac{1}{5}\right) - \arctan\left(\frac{1}{239}\right)$$

$$\text{So } \pi = 16 \arctan\left(\frac{1}{5}\right) - 4 \arctan\left(\frac{1}{239}\right)$$

$$\arctan\left(\frac{1}{x}\right) = \frac{1}{x} - \frac{1}{3x^3} + \frac{1}{5x^5} - \frac{1}{7x^7} + \dots$$

$$\text{So } 16 \arctan\left(\frac{1}{5}\right) = \frac{16}{5} - \frac{16}{3 \cdot 5^3} + \frac{16}{5 \cdot 5^5} - \frac{16}{7 \cdot 5^7} + \dots$$

$$\frac{16}{5} \div 25 \quad \frac{16}{5^3} \div 25 \quad \frac{16}{5^5} \div 25 \quad \frac{16}{5^7}$$

$$\text{and } 4 \arctan\left(\frac{1}{239}\right) = \frac{4}{239} - \frac{4}{3 \cdot 239^3} + \frac{4}{5 \cdot 239^5} - \frac{4}{7 \cdot 239^7} + \dots$$

$$\frac{4}{239} \div 239^2 \quad \frac{4}{239^3} \div 239^2 \quad \frac{4}{239^5}$$

$$+ \quad 3 \cdot 2$$

$$\begin{array}{r} 239 \\ 239 \\ 81 \\ 270 \\ 1800 \\ 270 \\ 900 \\ 6000 \\ 47800 \\ \hline 57121 \end{array}$$

$$\begin{array}{r} 57121 \\ 114242 \\ 171363 \\ 228434 \\ 285605 \\ 342726 \\ 399847 \\ 456568 \\ 514089 \\ \hline 571210 \end{array} \quad \begin{array}{r} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array} \quad \begin{array}{r} 239 \\ 478 \\ 717 \\ 956 \\ 1195 \\ 1434 \\ 1673 \\ 1912 \\ 2151 \\ \hline 2390 \end{array}$$

$$\begin{array}{r} 0.0167364 \\ 239 \overline{) 4.0000000} \\ \underline{239} \\ 1610 \\ \underline{1434} \\ 1760 \\ \underline{1673} \\ 870 \\ \underline{717} \\ 1530 \\ \underline{1434} \\ 960 \\ \underline{956} \\ 4 \end{array}$$

$$\begin{array}{r} 3.2000000 \\ 0.0167364 \\ \hline 3.183263598326359832635 \end{array}$$

or

$$3.18326359$$

$$\begin{array}{r} 0.128 \\ 25 \overline{) 3.2} \\ \underline{25} \\ 70 \\ \underline{50} \\ 200 \\ \underline{200} \end{array}$$

$$\begin{array}{r} 16 \\ \hline 3.5^3 \end{array} \quad \begin{array}{r} 0.0426 \\ 3 \mid 0.128^{20^2} \end{array}$$

$\frac{4}{239}$	57121	0.0 0 0 0 0 0 2 9 2 9 9 9 1 0 1 4 4 5 0 0 5 6 4 3 5 2 8 9 4 2 7 0
		1 0.0 1 6 7 3 6 4 0 1 6 7 3 6 4 0 1 6 7 3 6 4 0 1 6 7 3 6 4 0 1 6 7
		<u>1 1 4 2 4 2</u> 8 2 5 4 0 2 0 1 5 7 6
		5 3 1 2 2 0 5 7 1 2 1 1 7 1 3 6 3
		<u>5 1 4 0 8 9</u> 2 5 4 1 9 1 3 0 2 1 3 7
		1 7 1 3 1 1 2 2 8 4 8 4 2 8 5 6 0 5
		<u>1 1 4 2 4 2</u> 2 5 7 0 7 6 1 6 5 3 2 3
		5 7 0 6 9 6 2 2 8 4 8 4 1 1 4 2 4 2
		5 1 4 0 8 9 2 8 5 9 2 7 5 1 0 8 1 6
		5 6 6 0 7 7 2 8 5 6 0 5 4 5 6 9 6 8
		<u>5 1 4 0 8 9</u> 3 2 2 3 6 4 5 3 8 4 8 4
		5 1 9 8 8 3 2 8 5 6 0 5 5 1 4 0 8 9
		<u>5 1 4 0 8 9</u> 3 6 7 5 9 0 2 4 3 9 5 0
		5 7 9 4 6 3 4 2 7 2 6 2 2 8 4 8 4
		<u>1 5 7 1 2 1</u> 2 4 8 6 4 1 1 5 4 6 6 1
		8 2 5 4 2 2 8 4 3 4 1 1 4 2 4 2
		4 0 4 1 9 6
		<u>3 9 9 8 4 7</u>
		4 3 4 9 7

$$+ \frac{4}{3 \cdot 239} \quad 0.0.0000009766636714833521450964756$$

	3	1	8	3	2	6	3	5	9	8	3	2	6	3	5	9	8	3	2	6	3	5	9	8	3
+	0	0	0	0	0	0	0	9	7	6	6	6	3	6	7	1	4	8	3	3	5	2	1	4	5

[illegible]

3.1405970293260603143045311065792

$$\begin{array}{r} 0.00512 \\ 25 \overline{) 0.1280} \\ \underline{125} \\ 30 \\ \underline{25} \\ 50 \end{array}$$

	0	0	0	1	0	2	4
5	0	0	0	5	1	2	0

[illegible]

$$\begin{array}{r} 4 \\ - 5.2395 \end{array}$$

[illegible]

	3	1	4	1	6	2	1	0	2	9	3	2	6	0	6	0	3	1	4	3	6	4	5	3	1	1	0	6	5	7	9	2
-	0	0	0	0	0	0	0	0	0	0	0	0	1	0	2	5	8	8	9	2	5	7	6	9	8	5	8	9	4	6	2	8

301416210293250344250468325171164

$$\begin{array}{r} \text{—} \quad \overset{10}{2} \overline{) 57} \quad 7 \overline{) 0.0000292571428} \\ \underline{20} \quad \quad \quad \underline{70} \quad \quad \quad \underline{00} \quad \quad \quad \underline{00} \quad \quad \quad \underline{02} \quad \quad \quad \underline{04} \quad \quad \quad \underline{80} \quad \quad \quad \underline{50} \quad \quad \quad \underline{00} \quad \quad \quad \underline{20} \quad \quad \quad \underline{00} \quad \quad \quad \underline{04} \end{array}$$

$$\begin{array}{r}
 4 \\
 + \quad \quad \quad 7 \\
 \hline
 72357
 \end{array}$$

3. 1 4 1 6 2 1 0 2 9 3 12 5 0 3 4 4 3 7 8 7 5 3 5 5 1 4 8 2 5 5 2
- 0 0 0 0 0 2 9 2 5 7 1 4 2 3 5 7 1 4 2 8 5 7 1 4 2 3 5 7 1 4 2 9

$$\begin{array}{r} 16 \\ 9 \overline{) 9.59} \end{array} \quad \begin{array}{r} 0.00000009102 \\ 0.000000081920 \end{array}$$

$$\begin{array}{r} 4 \\ 239 \overline{) 95721} \end{array} \quad \begin{array}{r} 0.000000000000 \\ 0.000000000000 \end{array}$$

zeros omitted
for my sanity :)

$$- \frac{4}{9.239} \approx 0.433$$

0	0	0	0	1	5	7	2	0	9	5	3	4	8	7	8
8	9	7	9	9	6	5	3	4	1	7	9	7	1	4	5
5	7	1	2	1					4	4	8	6	3	4	
3	2	6	7	8	6				3	9	9	3	4	7	
2	3	5	6	0	5										
4	1	1	1	8	1	5									
3	9	9	8	4	7										
1	1	9	8	8	8										
1	1	4	2	4	2										
		5	4	4	6	4	1								
		5	1	4	0	8	9								
		3	0	5	5	2	7								
		2	8	5	6	0	5								
		1	9	9	2	2	9								
		1	7	1	3	6	3								
		2	7	8	6	6	7								
		2	2	8	4	8	4								
		5	0	1	8	3	1								
		4	5	6	9	16	8								

1	7	4	6	7	7	2	6	0	9	7		
1	5	7	2	5	9	5	3	5	4	8	7	8

[illegible]

3 1 4 1 5 9 2 6 8 2 4 0 4 3 9 9 5 1 7 2 4 0 4 3 4 5 1 3 3 3 4 5
1 7 4 6 7 7 2 6 1 0

2014 15 9 26 32 40 43 99 51 72 40 25 98 36 07 35

$\frac{16}{11.5''}$

$$+ \frac{-4}{11.234} \parallel 0.$$

2	5	0	2	0	1
2	7	5	2	2	1

	3	1	4	1	5	9	2	6	8	2	4	0	4	3	9	9	5	1	7	2	4	0	2	5	9	8	3	6	0	7	3	5	
+																													2	5	0	2	0

[illegible]

3.1415926526153086081493507476664

$$\begin{array}{r} 16 \\ 5 \overline{) 25} \end{array} \begin{array}{l} 0.00000003276800 \\ 25 \overline{) 50} \end{array}$$

$$\begin{array}{r} 77 \\ 75 \\ \hline 26 \\ 25 \\ \hline 180 \\ 175 \\ \hline \end{array}$$

$$\begin{array}{r} 1381 \\ 262 \\ 393 \\ 524 \\ 655 \\ 786 \\ 917 \\ 1048 \\ 1179 \end{array}$$

$$+ \frac{16}{3.5^{13}} 13 | 0.$$

$$\begin{array}{r} 10082461538 \\ 131072000000000000 \end{array}$$

$$\frac{4}{259^{13}} 57121 | 0.$$

$$\begin{array}{r} 2752219 \\ 278484 \\ \hline 467379 \end{array}$$

$$- \frac{4}{13-239^{13}} 13 | 0.$$

Ignore

$$\begin{array}{r} 3 \\ 48 \\ \text{Negligible for} \\ \text{30 digits} \end{array}$$

$$\frac{16}{5^{15}} 25 | 0.$$

$$\begin{array}{r} 524288 \\ 1310720 \\ 125 \quad 200 \\ \hline 60 \\ 50 \\ \hline 107 \\ 100 \\ \hline 72 \\ 50 \\ \hline 220 \\ 200 \\ \hline \end{array}$$

$$\begin{array}{r} 151 \\ 302 \\ 453 \\ 604 \\ 755 \\ 906 \\ 1057 \\ 1208 \\ 1359 \end{array}$$

$$- \frac{16}{15.5^{15}} 15 | 0.$$

$$\begin{array}{r} 3495253 \\ 524288000000000000 \end{array}$$

$$\frac{16}{5} \overline{) 250}$$

$$\begin{array}{r} 33554432 \\ 8388608 \\ \hline 75 \\ 88 \\ \hline 75 \\ 138 \\ \hline 125 \\ 136 \\ \hline 125 \\ 110 \\ \hline 100 \\ 108 \\ \hline 100 \end{array}$$

$$\begin{array}{r} 21 \\ 42 \\ 63 \\ 84 \\ 105 \\ 126 \\ 147 \\ 168 \\ 189 \end{array}$$

$$+ \frac{16}{21 \cdot 5} \overline{) 2100}$$

$$\begin{array}{r} 159783009523809 \\ 33554432000000000 \\ \hline 21 \\ 125 \\ 105 \\ 205 \\ \hline 189 \\ 164 \\ \hline 147 \\ 174 \\ \hline 168 \\ 263 \\ \hline 63 \end{array}$$

$$\begin{array}{r} 009523809 \\ 2000000000000 \\ \hline 200 \\ 189 \\ \hline 110 \\ 105 \\ \hline 50 \\ 42 \\ \hline 80 \\ 163 \\ \hline 170 \\ 168 \end{array}$$

$$\frac{16}{523} \overline{) 250}$$

$$\begin{array}{r} 134217728 \\ 3355443200 \\ \hline 25 \\ 85 \\ \hline 75 \\ 105 \\ \hline 100 \\ 54 \\ \hline 50 \\ 44 \\ \hline 25 \end{array}$$

$$\begin{array}{r} 23 \\ 46 \\ 69 \\ 92 \\ 115 \\ 138 \\ 161 \\ 184 \\ 207 \end{array}$$

$$- \frac{16}{23 \cdot 5} \overline{) 2300}$$

$$\begin{array}{r} 5335553391304347 \\ 134217728000000000 \\ \hline 115 \\ 192 \\ \hline 184 \\ 81 \\ \hline 69 \\ 127 \\ \hline 115 \\ 127 \\ \hline 115 \\ 122 \\ \hline 115 \\ 178 \\ \hline 69 \\ 190 \\ \hline 69 \end{array}$$

$$\begin{array}{r} 30 \\ 23 \\ \hline 70 \\ 69 \\ \hline 100 \\ 92 \\ \hline 80 \\ 69 \\ \hline 110 \\ 92 \\ \hline 180 \end{array}$$

$$\frac{16}{25} \div \frac{25}{25} = 0.$$

$$\begin{array}{r} 536870912 \\ 134217728 \\ \hline 125 \\ 92 \\ 75 \\ \hline 171 \\ 150 \\ \hline 217 \\ 200 \\ \hline 177 \\ 175 \\ \hline 2 \end{array}$$

$$+ \frac{16}{25} \div \frac{25}{25} = 0.$$

$$\left(= \frac{16}{25} \right)$$

$$\begin{array}{r} 2147483648 \\ 53687091200 \\ \hline 50 \\ 36 \\ 25 \\ \hline 118 \\ 100 \\ \hline 187 \\ 175 \\ \hline 120 \\ 100 \\ \hline 100 \end{array}$$

$$\begin{array}{r} 27 \\ 54 \\ 81 \\ 108 \\ 135 \\ 162 \\ 189 \\ 216 \\ 243 \end{array}$$

$$\begin{array}{r} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array}$$

$$- \frac{16}{27} \div \frac{27}{27} = 0.$$

$$\begin{array}{r} 79536431407 \\ 21474836480000 \\ \hline 189 \\ 257 \\ 243 \\ \hline 144 \\ 135 \\ \hline 98 \\ 81 \\ 173 \\ 162 \\ \hline 116 \\ 108 \end{array}$$

$$\begin{array}{r} 84 \\ 81 \\ 38 \\ 27 \\ \hline 110 \\ 108 \\ \hline 200 \\ 189 \\ \hline 110 \end{array}$$

	8	5	8	9	9	3	4	5	9	2		
2	1	4	7	4	8	3	6	4	8	0	0	29
<u>2</u>	<u>0</u>					8	6					58
	1	4	7			<u>7</u>	5					87
	<u>1</u>	<u>2</u>	5			1	1	4				116
		2	2	4		<u>1</u>	<u>0</u>	0				145
		<u>2</u>	<u>0</u>	0			1	4	8			174
			2	4	8		<u>1</u>	<u>2</u>	5			203
			<u>2</u>	<u>2</u>	5			2	3	0		232
				2	3	3		<u>2</u>	<u>2</u>	5		261
				<u>2</u>	<u>2</u>	5				5	0	
					8							

Handwritten long division on grid paper:

1. $29620 \overline{) 858993}$
 58
 278
 261
 179
 174
 59
 58
 134
 116
 185

2. $4641103 \overline{) 459200}$
 185
 174
 116
 32
 29
 30
 29
 100

	3	4	3	5	9	7	3	8	3	6	8
8	5	8	9	9	3	4	5	9	2	0	0
<u>75</u>						<u>95</u>					
1	0	8				<u>75</u>			3	1	1
<u>100</u>						209			6	2	2
	8	9				<u>200</u>			9	3	3
	<u>75</u>					92	1	2	4	4	
	1	4	9			<u>75</u>	1	5	5	5	
	<u>125</u>					<u>170</u>	1	8	6	6	
		2	4	3		<u>150</u>	2	1	7	7	
		<u>2,25</u>				20	2	4	8	8	
			1	8	4		2	7	9	9	
		<u>1,75</u>									
						<u>95</u>					

$$\begin{array}{r}
 11083786570 \\
 343597383620 \\
 \hline
 31 \\
 33 \\
 31 \\
 \hline
 259 \\
 248 \\
 \hline
 117 \\
 193 \\
 \hline
 243 \\
 217 \\
 \hline
 268 \\
 242 \\
 \hline
 203 \\
 186
 \end{array}$$

$$\frac{16}{5} \cdot 33 \quad 25 \mid 0.$$

33	1
66	2
99	3
132	4
165	5
198	6
231	7
264	8
297	9

$$+ \frac{16}{33 \cdot 5} \cdot 33 \mid 0.$$

$$\frac{16}{5} \cdot 35 \quad 25 \mid 0.$$

35	1
70	2
105	3
140	4
175	5
210	6
245	7
280	8
315	9

$$- \frac{16}{35 \cdot 5} \cdot 35 \mid 0.$$

$$\begin{array}{r} 137438953472 \\ 3435973836800 \\ \hline 25 \\ 93 \\ 75 \\ 185 \\ 175 \\ 109 \\ 100 \\ 97 \\ 75 \\ 223 \\ 200 \end{array} \quad \begin{array}{r} 238 \\ 225 \\ 133 \\ 125 \\ 86 \\ 75 \\ 118 \\ 100 \\ 180 \\ 175 \\ 50 \end{array}$$

$$\begin{array}{r} 4164816771 \\ 137438953472 \\ \hline 132 \\ 54 \\ 33 \\ 213 \\ 198 \\ 158 \\ 132 \\ 269 \\ 264 \end{array} \quad \begin{array}{r} 55 \\ 33 \\ 213 \\ 198 \\ 254 \\ 231 \\ 237 \\ 231 \\ 62 \end{array}$$

$$\begin{array}{r} 5497558138 \\ 137438953472 \\ \hline 125 \\ 124 \\ 100 \\ 243 \\ 225 \\ 188 \\ 175 \\ 139 \\ 125 \\ 145 \\ 125 \end{array} \quad \begin{array}{r} 203 \\ 200 \\ 34 \\ 25 \\ 97 \\ 75 \\ 222 \end{array}$$

$$\begin{array}{r} 157073089 \\ 5497558138 \\ \hline 35 \\ 199 \\ 175 \\ 247 \\ 245 \\ 255 \\ 245 \\ 108 \\ 105 \\ 313 \\ 280 \\ 338 \end{array}$$

$$\frac{16}{5} 37 \quad 25 \mid 0$$

3	7	1
7	4	2
1	1	3
1	4	4
1	8	5
2	2	6
2	5	7
2	9	8
3	3	9

$$+ \frac{16}{37.5}$$

$$\frac{16}{5} 39 \quad 25 \mid 0$$

3	9	1
7	8	2
1	1	3
1	5	4
1	9	5
2	3	6
2	7	7
3	1	8
3	5	9

2	1	9	9	0	2	3	2	5
5	4	9	7	5	5	8	1	3
5	0				5	8		
4	9				5	0		
2	5				8			
2	4	7			7	5		
2	2	5					6	3
2	2	5					5	0
2	2	5					1	3

5	9	4	3	3	0	6
2	1	9	9	0	2	3
1	8	5				2
3	4	9				
3	3	3				
1	6	6				
1	4	8				
1	2	2				
1	1	1				
1	1	3				
1	1	1				

8	7	9	6	0	9	3
2	1	9	9	0	2	3
2	0	0			2	3
1	9	9			2	2
1	7	5				7
2	4	0				
2	2	5				
1	5	2				
1	5	0				
2						

2	2	5	5	4	0
8	7	9	6	0	9
7	8				
9	9				
7	8				
2	1	6			
1	9	5			
2	1	0			
1	9	5			
1	5	9			
1	5	6			
3	3				

$$\begin{array}{r} 16 \\ 5 \overline{) 41} \end{array} 25 \mid 0.$$

4	1	1
8	2	2
1	2	3
1	6	4
2	0	5
2	4	6
2	8	7
3	2	8
3	6	9

$$+ \begin{array}{r} 16 \\ 41 \cdot 5 \overline{) 41} \end{array} 41 \mid 0.$$

$$\begin{array}{r} 16 \\ 5 \overline{) 43} \end{array} 25 \mid 0.$$

4	3	1
8	6	2
1	2	9
1	7	2
2	1	5
2	5	8
3	0	1
3	4	4
3	8	7

$$- \begin{array}{r} 16 \\ 43 \cdot 5 \overline{) 43} \end{array} 43 \mid 0.$$

$$\begin{array}{r} 16 \\ 5 \overline{) 45} \end{array} 25 \mid 0.$$

4	5	1
9	0	2
1	3	5
1	8	0
2	2	5
2	7	0
3	1	5
3	6	0
4	0	5

$$+ \begin{array}{r} 16 \\ 45 \cdot 5 \overline{) 45} \end{array} 45 \mid 0.$$

$$\begin{array}{r} 16 \\ 5 \overline{) 47} \end{array} 25 \mid 0.$$

$$- \begin{array}{r} 16 \\ 47 \cdot 5 \overline{) 47} \end{array} 47 \mid 0.$$

$$\begin{array}{r} 351835 \\ 8796093 \\ 75 \quad 89 \\ 129 \quad 75 \\ 125 \quad 143 \end{array}$$

$$\begin{array}{r} 46 \\ 25 \\ 210 \\ 200 \end{array}$$

$$\begin{array}{r} 8581 \\ 351835 \\ 328 \\ 238 \\ 205 \\ 333 \\ 328 \\ 55 \end{array}$$

$$\begin{array}{r} 14073 \\ 351835 \\ 25 \\ 101 \\ 100 \\ 183 \\ 175 \\ 85 \end{array}$$

$$\begin{array}{r} 327 \\ 14073 \\ 129 \\ 117 \\ 86 \\ 313 \end{array}$$

$$\begin{array}{r} 562 \\ 14073 \\ 125 \\ 157 \\ 150 \\ 73 \end{array}$$

$$\begin{array}{r} 12 \\ 562 \\ 45 \\ 112 \end{array}$$

$$\begin{array}{r} 21 \\ 562 \\ 50 \\ 62 \end{array}$$

$$\begin{array}{r} 0 \\ 21 \end{array}$$

	3	.	1	4	1	5	9	2	6	5	2	6	4	5	3	0	8	6	0	2	1	4	9	3	5	0	2	4	7	6	6	6	4			
+														1	0	0	8	2	4	6	1	5	3	8	4	6	1	5	3	8	4	6	1	5	3	8
	3	.	1	4	1	5	9	2	6	5	3	6	2	3	5	5	4	7	6	1	9	9	5	5	0	4	5	9	3	8	2	0	2			
-														3	4	9	5	2	5	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3		
	3	.	1	4	1	5	9	2	6	5	3	5	8	8	6	0	2	2	2	8	6	6	2	1	7	1	2	6	0	4	8	6	9			
+														1	1	2	3	3	6	1	8	8	2	3	5	2	9	4	1	1	7	6	4	7		
	3	.	1	4	1	5	9	2	6	5	3	5	8	9	8	3	5	8	4	7	4	8	5	7	0	0	6	7	2	2	5	1	6			
-																																				
	3	.	1	4	1	5	9	2	6	5	3	5	8	9	7	9	1	6	9	6	9	1	7	2	7	9	6	1	9	5	2	0	0			

3.1415926535897916969172796196200
 + 15978300952380952

3.1415926535897932947473748572152
 - 5523555339130435

3.1415926535897932363912407446717
 + 2147483648

3.1415926535897932385393245926717
 - 795364314074

3.1415926535897932384597881612643
 + 29620464170

29

3.1415926535897932384627502076753
 - 1108378657

31

3.1415926535897932384626393698096
 + 41648168

32

3.1415926535897932384626435346264
 - 11570731

35

3.1415926535897932384626433775533
 + 59433

37

3.1415926535897932384626433834966
 - 2255

39

3.1415926535897932384626433832711
 + 86

41

797
 - 3

43

794
 - 0

45

3.1415926535897932384626433232794

3.1415926535897932384626433832795

— Correct to 30 dp